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Coupling constants for the cosmic ray muon component

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Abstract. An apparatus employing twin telescopes to study the azimuthal variation of cosmic ray muon intensity was operated at Ahmedabad (geographic 23°N, 73°E; geomagnetic latitude 14°N). Fourteen directions were scanned for long time intervals to give a counting rate accuracy of about 0·4% for each direction. Experimental results are presented. Variation in the coupling constant values in the rigidity range 12–26 GV is rather flat. Some implications of this behaviour are discussed.

A theoretical method is described for the estimation of coupling constants applicable to muon telescopes. Correction for the fact that alpha particles and heavy particles have a cut-off (in terms of momentum per nucleon) at half the value for protons is incorporated in our calculation. The method is verified by comparing the theoretical results with experimental data on the latitude effect (namely, for zenith direction) and on the azimuth effect for fixed zenith angles. Theoretically estimated coupling constants for different zenith angles and different atmospheric depths are presented.

1. Introduction

Dorman's coupling constants play a vital role in connecting the changes of the cosmic ray secondaries with those of the cosmic ray primaries. If $N_\lambda^i(h)$ represents the intensity of a secondary component of type i at a latitude λ and at an atmospheric depth h and if the primary spectrum is represented by $D(E)$ and $M^i(E, h)$ represents the multiplicity function that gives the number of secondaries of type i produced at an atmospheric depth h due to a primary particle of rigidity E; then the coupling constant $W_\lambda^i(E, h)$ is given by

$$W_\lambda^i(E, h) = \frac{D(E)M^i(E, h)}{N_\lambda^i(h)} \tag{1}$$

so that

$$\int_{E_\lambda^c}^{\infty} W_\lambda^i(E, h)\,\mathrm{d}E = 100\% \tag{2}$$

where E_λ^c is the cut-off rigidity for the arrival of primaries at a latitude λ. Thus, the coupling constant applicable for any detector gives the percentage contribution of primaries of a given rigidity to the counting rate of secondary cosmic rays of a given type.

Coupling constants may be calculated on purely theoretical grounds provided the theoretical model giving details of the nuclear interactions between primary cosmic rays and atmospheric nuclei, for the estimation of the multiplicity function $M^i(E, h)$, is accurately known. Uncertainties in the estimation of the multiplicity function have led to empirical derivations of the coupling constants (Dorman 1957, Webber and Quenby 1959, Dorman *et al* 1970, Cooke and Fenton 1971). Coupling constants can be obtained empirically from geomagnetic effects (latitude effects) for the vertical intensity at sea level as well as at high altitudes where experimental data are available as follows (Dorman 1957)

$$W^i_\lambda(E^c_{\lambda'}, h) = -\frac{1}{N^i_\lambda(h)} \frac{\delta N^i_{\lambda'}(h)}{\delta E^c_{\lambda'}} \tag{3}$$

where $N^i_{\lambda'}(h)$ is the intensity of a secondary component of type i at a latitude λ' and $E^c_{\lambda'}$ is the corresponding cut-off rigidity at that place, the negative sign appearing because an increase in cut-off rigidity ($\delta E^c_{\lambda'}$) results in a decrease ($\delta N^i_{\lambda'}(h)$) in the intensity thus giving a negative rate of change. However, in this way, coupling constants can be obtained only up to about 15 GV rigidity which is the maximum cut-off rigidity for vertical intensity at the equator. Coupling constants for higher rigidities can then be extrapolated by the method suggested by Dorman (1957).

Kane (1962) pointed out the discrepancy in the results of Dorman (1957) and Webber and Quenby (1959) and attributed it to the extrapolation methods. He showed the importance of azimuth effect data for providing experimental observations beyond 15 GV rigidity. The cut-off rigidity is given by

$$E^c_\lambda = 59{\cdot}6 \frac{\cos^4 \lambda}{[1 + (1 - \sin\theta \cos\phi \cos^3 \lambda)^{1/2}]^2} \text{ GV} \tag{4}$$

where θ is the zenith angle and ϕ is the azimuth angle measured from the north. Thus with the help of a telescope inclined to zenith at about 50° at geomagnetic equator and taking counting rates at different azimuths, coupling constants in the range 10–30 GV can be obtained.

Coupling constants for the vertical and inclined directions at the same location are not directly comparable, as the latter refer to a greater atmospheric depth and hence will have a flatter coupling constant against rigidity plot, the maximum value of the coupling constant occurring at the highest rigidities. However, measurements for inclined directions (azimuth effect data) will at least give a clue to the shape of the response curve for the rigidity region 10–30 GV. Also coupling constants for inclined directions at higher altitudes could be compared with those of vertical intensity at sea level, as both would involve roughly similar atmospheric paths.

Kane (1962) calculated coupling constants from the data of various azimuthal surveys carried out near the equator and reported in the literature up to 1961 and showed that the experimental data then available were not sufficiently accurate for this purpose and there was a need for more accurate azimuth effect data.

2. Experimental arrangement

An apparatus, consisting of two independent triple coincidence GM counter telescopes each of semi-angles 18° × 22°, was operated at Ahmedabad (geographic 23°N, 72·6°E; geomagnetic latitude 14°N) during the period 1967–69 to study the azimuthal variation of cosmic ray meson intensity. The zenith angle was fixed at 45° and the two telescopes pointed to directions 180° apart in azimuth. The arrangement was capable of rotating about a vertical axis. Thus the telescopes could be kept at any desired azimuth angle.

Each telescope consisted of three trays of counters. Six GM counters, each of diameter 4 cm and length 30 cm, formed a tray. An absorber of 15 cm of iron plates was interposed between the middle and bottom trays to cut off the soft component. The vertical distance between successive trays was 37 cm. Figure 1 shows the arrangement of the apparatus. The satisfactory operation of the apparatus was judged by comparing the results obtained in the same direction with two independent telescopes having the same geometry. In all, fourteen azimuths were scanned. Generally observations extending over a period of two to three months were found sufficient to obtain a counting rate accuracy of about 0·4 % for each direction.

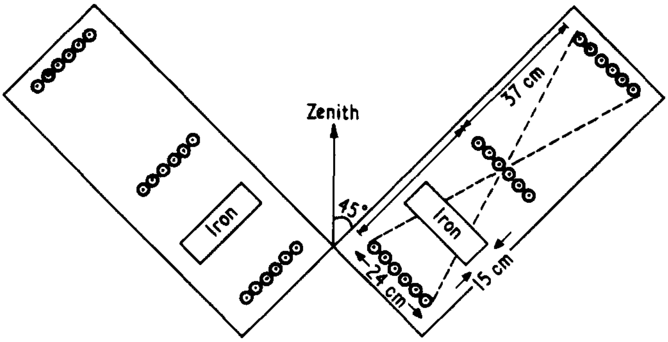

Figure 1. Apparatus for the study of the azimuthal variation of cosmic ray meson intensity.

3. Experimental data and analysis

Counting rates thus obtained for 14 azimuths and for both the telescopes were normalized to a 100 % level for the west direction. Thus we obtained normalized rates and standard errors for each of the fourteen directions and for both the telescopes.

Since the cosmic ray intensity decreases with zenith angle, the effective direction of the telescope will not be the same as the geometrical axis of the telescope. This difference depends on the opening angle of the telescope as well as on the variation of intensity with the zenith angle. Kane and Rao (1960) have shown that for a telescope inclined to the vertical at 45° and having a semi-angle 20°, the mean inclination of all the radiation recorded is at 42·5° and 50 % of the recorded radiation is incident within a range of $\pm 5{\cdot}5°$ of this mean value.

Daniel and Stephens (1966) have calculated the threshold rigidities by a sixth degree simulation of the geomagnetic field. They have kindly provided us threshold rigidities at Ahmedabad at every 10° interval in zenith and every 15° interval in azimuth. From these values threshold rigidities corresponding to the various azimuthal angles scanned by us for a fixed 42·5° zenith angle were obtained.

4. Results and discussion

Normalized rates obtained (as explained in the previous section) are shown in figure 2 as a function of azimuth. Two points for each azimuth refer to measurements taken with two independent telescopes. The error flags show one standard deviation. It can be seen from this figure that the rates recorded by the two telescopes corresponding to each azimuth are similar within the statistical limits for most of the azimuths studied, indicating thereby that both the telescopes worked satisfactorily during the whole period of

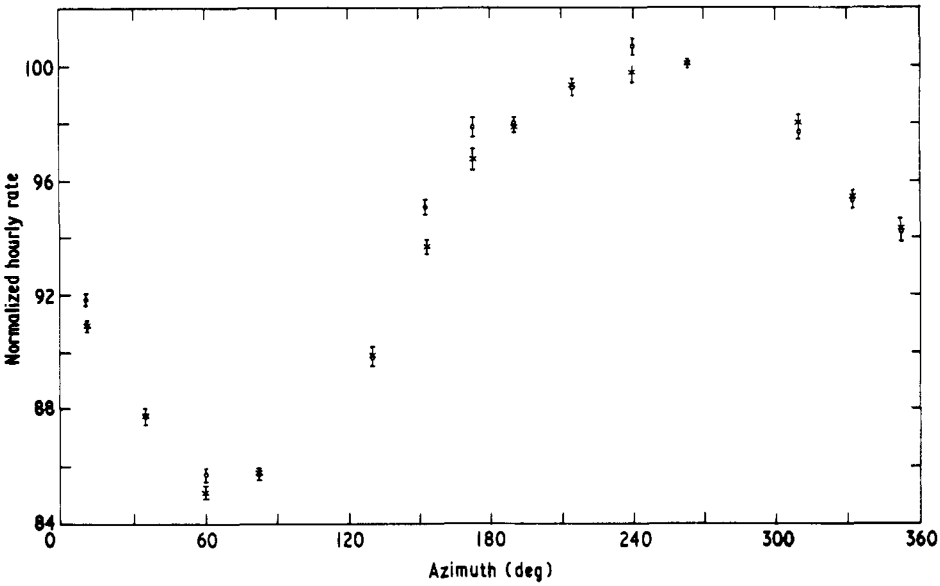

Figure 2. Azimuthal effect recorded by the GM counter telescopes at Ahmedabad for 42·5° zenith angle. The two points for each azimuth refer to observations taken with two independent telescopes and the error flags show one standard deviation.

operation. The normalized rate plotted against the threshold rigidity is shown in figure 3, where the rates of both telescopes are shown with their standard deviations.

The normalized rates of both telescopes for each azimuth are combined using the weighted mean method to obtain the normalized intensity as a function of rigidity. Also,

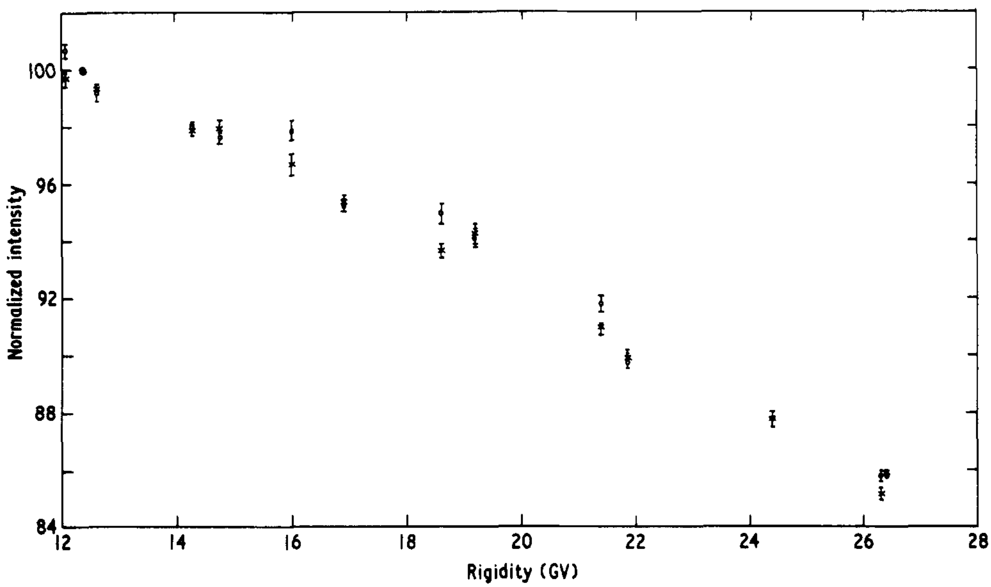

Figure 3. Meson intensity as a function of threshold rigidity at Ahmedabad as obtained by twin telescopes for an effective zenith angle of 42·5°.

for some points the rigidities were very close to each other and therefore weighted means of rigidities as well as intensities were taken for these points. Mean normalized intensities thus obtained are shown in figure 4 as a function of mean rigidity.

Coupling constants were then derived using equation (3) and are shown in figure 5 for the 12–26 GV rigidity range. The variation in coupling constant values is rather small in the whole range 12–26 GV. Some possible sources of inaccuracy which must be kept in mind are:

(i) *Opening angle of the telescope.* Coupling constants are derived using cut-off rigidities corresponding to a fixed zenith angle of 42·5° and appropriate azimuthal angles, but the finite opening angle of the telescope allows primaries to come from different directions centred around the mean direction. A very narrow angle telescope is therefore ideal for such accurate estimations. However, the counting rates would then be smaller and consequently counting rate accuracy would not be better unless very large areas and a very long period of operation are used.

(ii) *Contributions from heavier nuclei.* In deriving the coupling constants we have used the cut-off rigidity values pertaining to protons incident on the top of the atmosphere in the respective directions. However, at any latitude, nearly 26% of the primary nucleons are alpha particles and heavy particles and hence do not have the cut-off at a value corresponding to that of primary proton but at only half of that value (in terms of momentum per nucleon). This also needs to be taken into account for an accurate estimation of the coupling constants. We have taken this into account in the theoretical estimation of the coupling constants applicable for muon monitors.

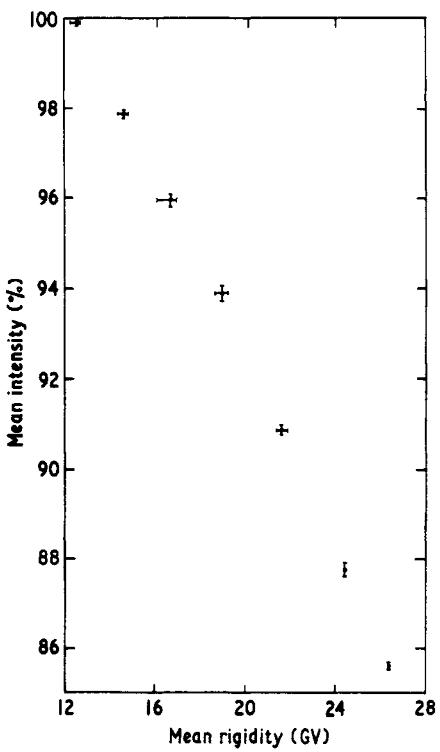

Figure 4. Meson intensity as a function of rigidity recorded at Ahmedabad.

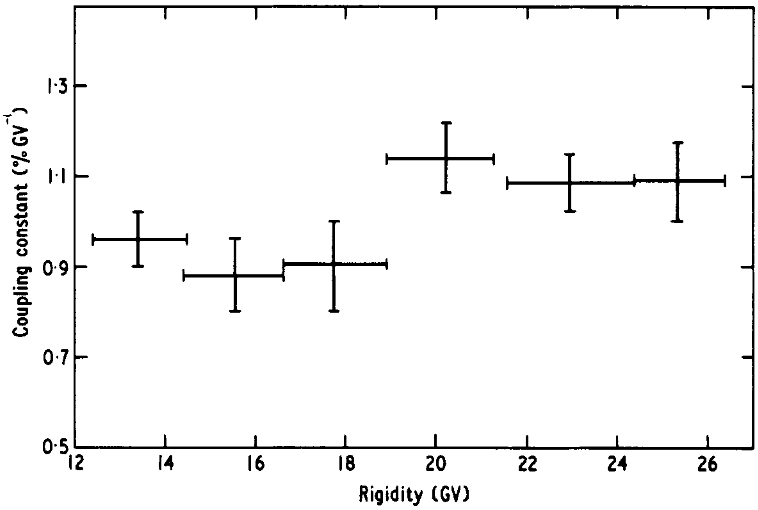

Figure 5. Coupling constants for the meson component, as derived from the azimuthal effect recorded at Ahmedabad as a function of rigidity for the 42·5° zenith angle.

5. Theoretical method of calculation of coupling constants

There have been various attempts in the past to obtain the coupling constants for muon monitors theoretically (Krimsky *et al* 1965, Peacock 1970, Ahluwalia and Ericksen 1971, Simpson and Mathews 1972). However, coupling constants obtained by these workers differ considerably from each other.

Our aim was to obtain coupling constants theoretically for several zenith angles and several atmospheric depths and compare them with the existing experimental data. For this, we made use of the model proposed by Åström (1966) which in turn was based on the earlier results of Bradt *et al* (1950) and Brunberg (1958). In this model the energy spectrum of the vertically incident muons is calculated by considering the various processes in the atmosphere. In the low energy range, which accounts for the bulk of the recorded intensity at sea level, Åström obtained good agreement with the measured muon spectra by fitting some parameters. We have made appropriate changes in Åström's model to calculate the muon flux at inclined directions as well and have also incorporated the correction for the contribution of heavy particles.

To obtain coupling constants applicable to a muon monitor operated at a given depth and in a given direction we should obtain the relative contribution of primaries of different rigidities to the counting rate of the detector involved. The integral intensity of muons, N_μ, at a particular place is obtained by integrating the muon differential energy spectrum over all muon energies. Thus,

$$N_\mu = \int_{E_{p1}}^{E_{p2}} dE_p \int_{E_{\mu 1}}^{E_{\mu 2}} dE_\mu \int_0^{\pi/2} d\alpha_p \int_0^{2\pi} d\phi_p \int_0^{x_0} dx \int_{E_{\pi 1}}^{E_{\pi 2}} \Psi(E_p, E_\mu, \alpha_p, \phi_p, x, E_\pi)\, dE_\pi \tag{5}$$

where $E_{\pi 1}$ and $E_{\pi 2}$ are the limits of pion energies capable of contributing to the muon flux at a given muon energy. x_0 is the air mass above sea level (or the atmospheric depth at which radiation is recorded) in g cm^{-2}. α_p and ϕ_p are the zenith and azimuth angles of the primary beam respectively and $E_{\mu 1}$ is the minimum energy required by a muon to traverse the telescope absorber.

Ψ is a complicated function of various parameters which represents the various terms as explained in detail by Åström (1966). N_μ cannot be obtained analytically and therefore the numerical integration approach has been adopted to evaluate the counting rates. Since secondary particles retain the direction of the primaries in most of the cases E_{p1} was chosen as the cut-off rigidity corresponding to the secondary direction (α_s, ϕ_s).

E_{p2} and $E_{\mu 2}$ in equation (5) are the upper limits of integration for primary ridigity and muon energy. In principle, these values should be infinity. However, in the numerical integration approach the integration has to be terminated at a suitable place (without introducing appreciable errors), otherwise the results for higher values of E_p and E_μ would be weighted more and the coupling constant results for low rigidities would largely be in error. We have chosen these upper limits as 1000 GV (E_{p2}) and 50 GeV ($E_{\mu 2}$). $E_{\mu 1}$ is generally of the order of 0·4 GeV and the muon contribution beyond 50 GeV is about 0·6 % (Hayman and Wolfendale 1962). Therefore, the integral intensity (N_μ) would be underestimated by about 0·6 % (by taking $E_{\mu 2} = 50$ GeV) and hence coupling constants might be overestimated by about 0·6 %. Primaries of momentum greater than 1000 GeV/c do not contribute more than 4 or 5 % at sea level for zenith angles of about 50° (Cooke and Fenton 1971). This contribution decreases both for telescopes having zenith angles less than 50° and for measurements at higher altitudes. Therefore, the integral muon intensity (N_μ) would, at most, be underestimated by 5 % by choosing E_{p2} as 1000 GV and hence the coupling constants might be overestimated by at most 5 %. Thus, both these effects (terminating integration at $E_{p2} = 1000$ GV and $E_{\mu 2} = 50$ GeV) might introduce an overestimate of about 6 % for the coupling constants.

In our calculations the primaries are assumed to interact once with the atmospheric nuclei and this is equivalent to replacing the nucleonic cascade with a single interaction process. Thus, the effective interaction mean free path (120 g cm^{-2}) concept has been introduced to account for the contributions from those primaries which interact more than once in the atmosphere. However, as recently pointed out by Groom and Morrison (1973) and Mathews and Hicks (1973) this assumption might give too much weight to the lower rigidities and thus result in a systematic shift in the coupling constant values at low rigidities.

To obtain the coupling constants, first $\mathrm{d}N_\mu/\mathrm{d}E_p$ is evaluated as a function of E_p. Then, the effective contribution from a given rigidity is obtained by

$$\left(\frac{\mathrm{d}N_\mu}{\mathrm{d}E_p}\right)_{\mathrm{eff}} = 0{\cdot}74\left(\frac{\mathrm{d}N_\mu}{\mathrm{d}E_p}\right)_{E_p} + 0{\cdot}26\left(\frac{\mathrm{d}N_\mu}{\mathrm{d}E_p}\right)_{E_p/2}. \tag{6}$$

The effective contributions thus obtained are then integrated from E_{p1} to E_{p2} (where E_{p1} and E_{p2} are chosen as explained earlier) to get the total muon flux for a narrow angle telescope located at a given place and looking in a particular direction. Coupling constants, corrected for the contribution of heavy particles are then obtained from equation (6) as a percentage of the total muon flux. The correction for the contribution of heavy particles is essential for: (i) calculating the latitude effect as obtained by latitude surveys; (ii) obtaining the azimuth effect (the east–west effect); and (iii) comparing the coupling constants obtained theoretically with those derived from experimental data.

6. Comparison of results with experimental data

We can compare our results of calculations up to 15 GV with the latitude effect data and up to 25 GV with the azimuth effect data for fixed zenith angles. The experimental data available so far for such comparisons are (*a*) the latitude effect data at sea level by Carmichael and Bercovitch (1969), (*b*) the azimuth effect data at sea level (Ahmedabad) at 42·5° zenith obtained by us, (*c*) the azimuth effect data at Makerere (1200 m above sea level) at 30° zenith by Mathews and Sivjee (1967), (*d*) the azimuth effect data at Timboroa (2700 m above sea level) at 43·5° zenith angle by Mathews and Sivjee (1967). The locations Makerere and Timboroa, where the azimuth effect studies (*c*) and (*d*) were carried out, were at such altitudes that the muons travelling at angles of 30° and 43·5° to the vertical, respectively, will traverse the same amount of air as vertically travelling muons at sea level.

Counting rates as a function of rigidity were calculated by the method described in the preceding section for each of the four above mentioned cases. Comparison of the calculated and experimental latitude and azimuth effects are shown in figures 6, 7 and 8. In figure 6 the predicted latitude effect at 312 $g\,cm^{-2}$ is also shown for comparison. In many cases there is a reasonable agreement between the predicted and observed effects, but in some cases the two seem to differ systematically, though not appreciably. We have evaluated the coupling constants applicable for meson monitors using equation (6), as explained earlier in § 5. In figure 9 the coupling constants are shown for meson monitors pointing to the zenith at 312 $g\,cm^{-2}$ (curve A) and at sea level (curve B). Both these curves are normalized so that at 5 GV rigidity the counting rate is 100.

In figure 10 theoretically calculated coupling constants are shown for a meson monitor at sea level (Ahmedabad) at a zenith angle of 45° (curve A) and at 1200 m above sea level at a zenith angle of 33° (curve B). Both these curves are normalized such that at 12 GV rigidity the counting rate is 100. In the same figure coupling constants derived

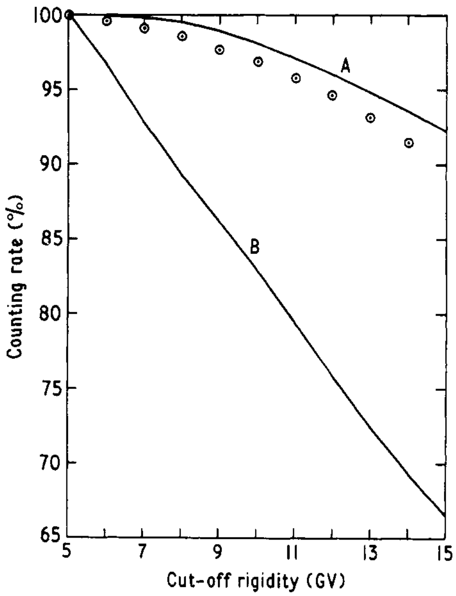

Figure 6. Comparison of calculated and observed latitude effects for the muon component. The full curves givc the calculated rates at sea level (curve A), and 312 $g\,cm^{-2}$ (curve B). The circles represent experimental values given by Carmichael and Bercovitch (1969) at sea level.

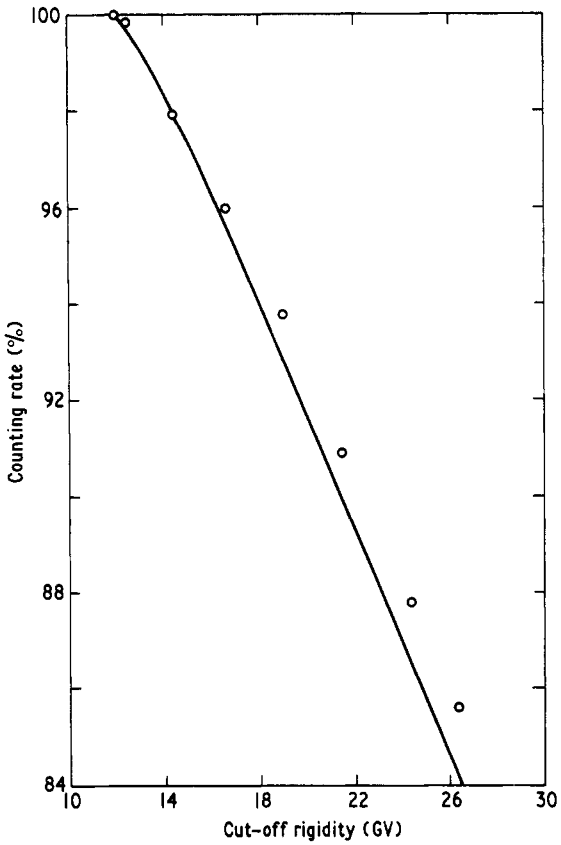

Figure 7. Comparison of the predicted and observed azimuthal effect at Ahmedabad for meson telescopes having an effective zenith angle of 42·5°. The full curve gives the calculated rates whereas the open circles represent experimental values.

from the azimuth effect experiment carried out at Ahmedabad are also shown for comparison. The coupling constants at a zenith angle of 47° and at 2700 m above sea level were also calculated and were found to be almost similar to those shown by curve B in figure 10. The decline of cosmic ray intensity with zenith angle and the finite opening angle employed in cosmic ray telescopes make the effective direction of the telescope different from the geometrical axis of the telescopes and hence effective directions 30°, 42·5° and 43·5° were used instead of the telescopic axis directions of 33°, 45° and 47° respectively.

7. Conclusions

An azimuth effect experiment was conducted at Ahmedabad to derive coupling constants. Counting rates were obtained for 14 directions well spread over all azimuths with a counting rate accuracy of about 0·4 %. Coupling constants derived from these rates did not show any appreciable variation with rigidity in the 12–26 GV rigidity range. However, the accuracy does not seem to be adequate to decide between Dorman's or Webber's

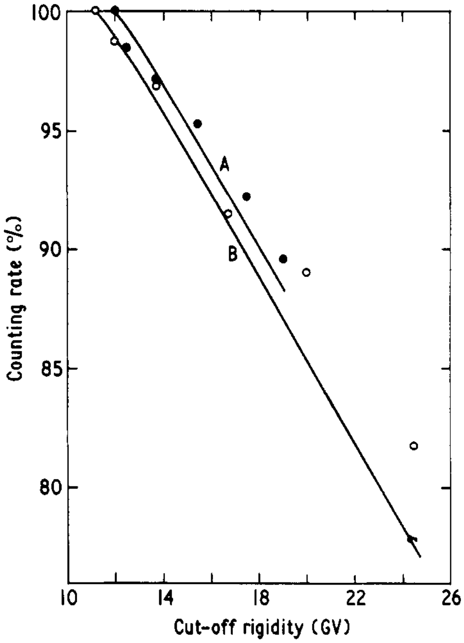

Figure 8. Comparison of the predicted and observed azimuth effects. The experimental points are those of Mathews and Sivjee (1967). Full circles represent values at 1200 m above sea level for 30° effective zenith angle and open circles represent values at 2700 m above sea level for an effective zenith angle of 43·5°. The curves A and B are the corresponding calculated curves.

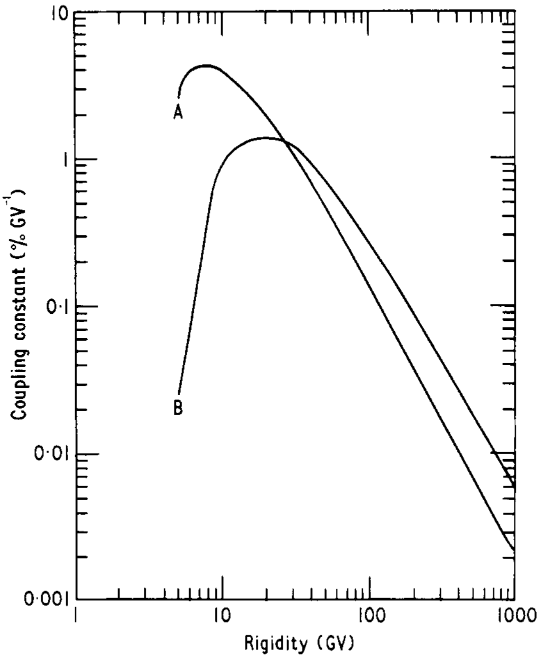

Figure 9. Theoretically derived coupling constants applicable to muon monitors pointing to zenith at 312 g cm^{-2} level (curve A) and sea level (curve B).

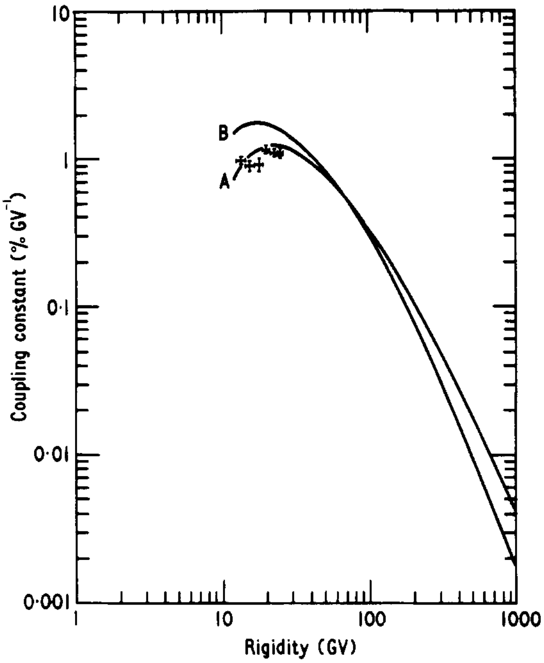

Figure 10. Theoretically derived coupling constants applicable to inclined muon telescopes at sea level at a zenith angle of 45° (curve A) and 1200 m above sea level at a zenith angle of 33° (curve B). Experimental points (with error bars) are obtained from the azimuth effect study at Ahmedabad.

curves. To resolve the discrepancy between the results of Dorman and Webber, much bigger telescopes than ours and/or much longer operational periods seem necessary.

A method incorporating the correction for the contribution of heavy particles is described for the theoretical estimation of the latitude effect, azimuth effect and coupling constants. Reasonably good agreement with the experimental data gives some justification to the method adopted. It is hoped that the method developed will be useful for many investigators in this field to calculate the coupling constants at various zenith angles and at various atmospheric depths. It is also hoped that the method would fulfil the long felt need for accurate estimates of the coupling constants, applicable to terrestrial muon telescopes, necessary for a proper interpretation and understanding of the phenomenon of time variation of the cosmic ray intensity.

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